Multi-Axis Motion System Error Budgeting

By: John Lindell

Application Engineering Manager

This article will explain how multi-axis motion systems can be analyzed to estimate functional point (work point) errors. Specifics on how to review system geometry and how to convert component-level errors into system-level errors will be discussed. A recommended calculation method, as well as ways to improve system performance, will also be examined.

Why Do Multi-Axis Errors Matter?

Modern manufacturing processes are generating increasingly complex part geometries and continually shrinking tolerances. Interventional medical devices, semiconductor chips and jet turbine blades are examples of products that require high levels of process and inspection tool performance to produce. This trend means machine builders need to have a deep understanding of component-level errors to build highly accurate manufacturing and test systems. Understanding these errors starts with knowing the tool or instrument's functional point error performance. Table 1 gives a few specific application examples, along with associated key performance requirements and the impact of these errors.

Table 1. *Examples of applications and the types of motion errors that impact process performance.*

Application	Which errors should be minimized?	Why does it matter?
Silicon wafer dicing	Horizontal straightness error at the functional (tool) point	Cutting inaccuracies lower process yield
Optics inspection	• Vertical straightness error over the measured part • Pitch errors of the linear stage • Rotational error motions of the rotary stage holding the part to be measured	Higher measurement uncertainty
Laser machining	Cut path error (ie. errors that result from not following the desired trajectory)	Part does not meet design specifications
Synchrotron (X-ray) inspection	Volumetric (3D) error, also referred to as sphere-of-confusion (many error sources contribute to this depending on the system's axis arrangement)	Blurred or inaccurate images of the measured part

In designing a machine, the feedback device (such as a laser interferometer or encoder) is necessary to measure the position of the moving axes. However, these devices have inherent errors that must be managed. In addition, parasitic motions of each moving axis are not necessarily detectable by the feedback device and contribute to the overall functional point error. As the number of moving axes increases in a machine, the total number of possible error sources also increases. Each component axis has the potential for six primary errors in each direction of motion: three linear and three rotational.

The applications in Table 1 illustrate some simple examples of how motion errors impact the output of the particular process. For the instrument designer, a detailed analysis of the motion platform's systematic errors provides important information on the tool's sensitivity and will ultimately help guide decisions that can lead to a better, more accurate design.

In an actual machine or instrument, motion errors are often only a portion of the machine's total error budget. Errors related to temperature, machine and/or part mounting, floor vibration, acoustic vibration and more also contribute to the overall error budget. This paper focuses only on the multi-axis *motion* error budget. However, the same approach used in this paper can be extended to other error sources to perform a complete error budget of the machine or instrument.

What Are the Motion Errors and How Are They Accounted for in an Error Budget?

In order to perform any error budget, a coordinate reference frame and definitions of each error relative to that reference frame must be established. Figure 1 shows the coordinate system with A, B and C used for rotations about the X-, Y- and Z-axes, respectively.

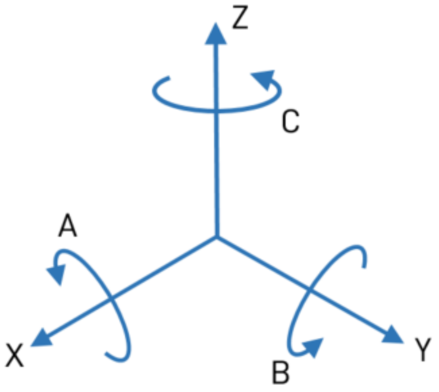

Figure 1. *Multi-axis error budget reference coordinate system.*

A generalized convention for describing an error in a motion system is

$$E_{IJ} = \textit{Error that occurs in the I-direction caused by the J-axis} \quad (1)$$

where *I* and *J* are generalized directions and axes. In this particular example, *I* and *J* could be X, Y, Z, A, B or C.

Table 2 shows the convention used to describe the errors that occur for a linear X-axis and a rotary C-axis.

Table 2. *Linear and rotary axis error conventions.*

Linear axis (X)		Rotary axis (C)	
Error	Description of error	Error	Description of error
E_{XX}	Linear position error of the X-axis in X-direction (accuracy)	E_{XC}	Radial error motion of the C-axis in the X-direction
E_{YX}	Horizontal straightness error of the X-axis in the Y-direction	E_{YC}	Radial error motion of the C-axis in the Y-direction
E_{ZX}	Vertical straightness error of the X-axis in the Z-direction	E_{ZC}	Axial error motion of the C-axis
E_{AX}	Angular error of the X-axis in the A-direction (roll)	E_{AC}	Tilt error motion of the C-axis in the A-direction
E_{BX}	Angular error of the X-axis in the B-direction (pitch)	E_{BC}	Tilt error motion of the C-axis in the B-direction
E_{CX}	Angular error of the X-axis in the C-direction (yaw)	E_{CC}	Angular position error of the C-axis in the C-direction (accuracy)

Now that a coordinate system and an error convention have been established, certain machine characteristics must be understood to perform an estimate of the multi-axis system error. These characteristics are as follows:

1) Machine configuration:
    a) How is the machine configured? (e.g. What are the total number of axes in the system that are directly associated with the measuring/manufacturing process?)
2) Axis configuration:
    a) How are the motion axes located relative to each other? (i.e. Are the axes stacked one on top of another or are they separated [e.g. split-axis]?)
3) Process point:
    a) Where is the functional point (e.g. tool point or work point), work plane or work volume located relative to the motion axes?
4) Process-sensitive direction:
    a) What direction(s) is the process sensitive to (e.g. X- and Y-directions in laser cutting, Z-direction in surface metrology, etc.)?
5) Process-critical errors:
    a) What are the component errors for all applicable degrees of freedom (DOF) for each axis, as well as inter-axis errors (e.g. orthogonality between X- and Y- axes)?

To illustrate the error budgeting process, an example application with two motion axes is shown in Figure 2. The example application is an inspection process of a 300 mm diameter part. The sensor is positioned so that the sensor measurement axis is located on the line segment that represents the diameter of the measured part.

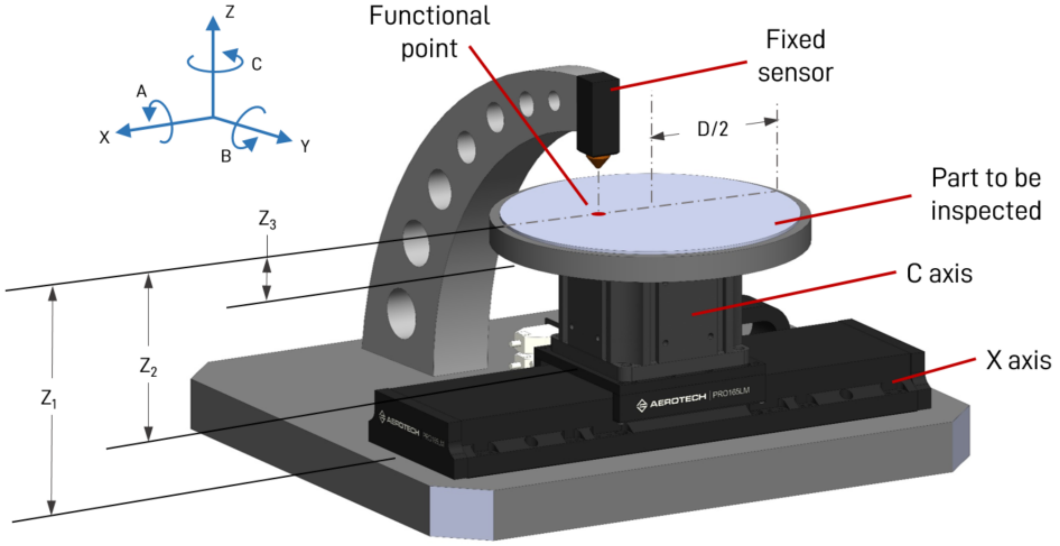

Figure 2. *Example system and application used for error budget analysis.*

Answering the five questions above helps to establish the critical errors that must be accounted for in the error budgeting process.

1) Machine configuration:
    a) Two-axis part inspection, moving part, fixed process head
2) Axis configuration:
    a) Stacked X-C axes with integrated chuck holding the 300 mm diameter part
3) Process point:
    a) The functional point (e.g. process point) is located on the top surface of the 300 mm diameter (D) part. The top surface is a distance of Z_1 above the stage mounting surface, a distance Z_2 from the X-axis tabletop and a distance Z_3 above the C-axis shaft mounting surface.
4) Process-sensitive direction:
    a) The sensor only measures displacement in the Z-direction and is "insensitive" to errors in the X- and Y-directions. Although errors may exist in X and Y, this example is only concerned with Z-direction errors.
5) Process-critical errors:
    a) Only errors that occur in the Z-direction are critical for this process. These errors are:
        i) E_{ZX} - Vertical straightness of the X-axis
        ii) E_{BX} - Pitch error of the X-axis
        iii) E_{ZC} - Axial error motion of the C-axis
        iv) E_{BC} - Tilt error motion of the C-axis in B-direction
        v) $E_{C\perp X}$ - Orthogonality (alignment) error of the C axis-of-rotation to the X-axis travel

b) Parallelism errors between the top surface of the chuck and rotary shaft mounting surface along with TIR of the shaft rotary surface will show up as similar "process errors" to the tilt error motion, except these errors occur at one cycle per revolution. For the purposes of this illustration, these errors are ignored.

The X-axis pitch error (E_{BX}), the C-axis tilt error motion (E_{BC}) and the orthogonality error between the C-axis and X-axis ($E_{C\perp X}$) all affect the measurement result since the part is measured at a varying radius. These errors are shown visually (exaggerated) in Figure 3.

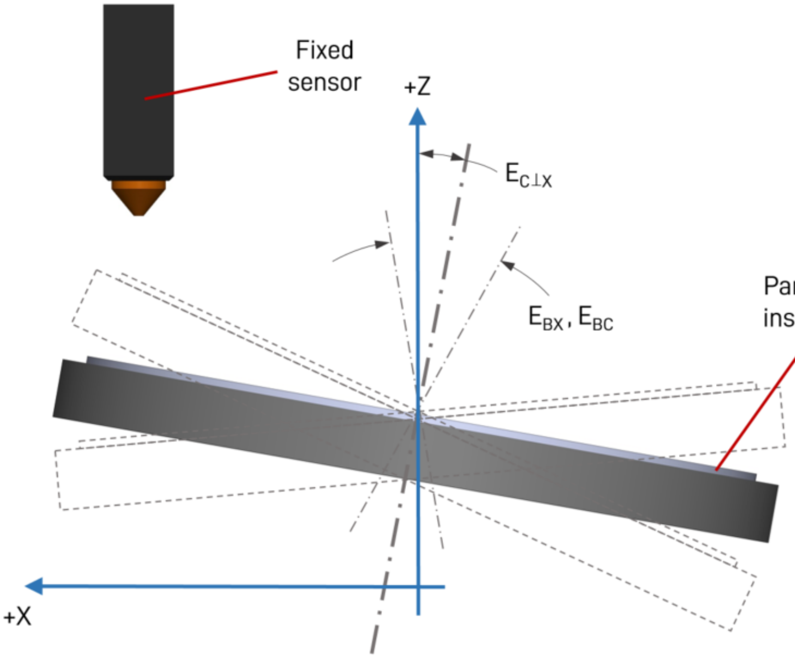

Figure 3. *Exaggerated illustration of X-axis pitch error* (E_{BX})*, C-axis tilt error motion* (E_{BC}) *and the orthogonality error between the C-axis and X-axis* ($E_{C\perp X}$)*.*

In order to perform the actual error budget with numerical values, the individual error values must be known. By using a variety of metrology techniques (outside the scope of this article), the errors can be measured and quantified. The values shown in Table 3 are used in this example.

Table 3. *Example dimensions and error values used in the error budget analysis.*

Symbol	Description	Value
D	Part diameter	300 mm
Z_1	Vertical distance from X-axis stage mounting surface to functional point	210 mm
Z_2	Vertical distance from X-axis tabletop to functional point	140 mm
Z_3	Vertical distance from C-axis shaft mounting surface to functional point	25 mm
E_{ZX}	Vertical straightness error of the X-axis	4 µm
E_{BX}	Pitch error of the X-axis	30 µrad
E_{ZC}	Axial error motion of the C-axis	2 µm
E_{BC}	Tilt error motion of the C-axis	15 µrad
$E_{C\perp X}$	Orthogonality (alignment) error between the X-axis and C-axis	25 µrad

How Should Errors Be Combined to Help Assess the System-Level Error?

The peak errors values, shown in Table 3, rarely add directly together in a real-world motion system. Although straight addition of the peak error values may provide a conservative estimate of the system-level error, using this approach will often result in an overspecified, costly motion system.

A more practical and often used approach is to add the errors in quadrature. Addition in quadrature is a mathematical operation used to combine two or more uncorrelated variables (in this case, errors). Using quadrature, the errors in the *I*-direction are calculated as

$$E_I = \sqrt{\sum_{K}^{N} (E_{IK})^2} \tag{2}$$

where *K* = the error that is contributing *I*-direction error and *N* = the number of errors for each axis that contributes to the *I*-direction error.

The vertical straightness error of the X-axis, E_{ZX}, and the axial error motion of the C-axis, E_{ZC}, contribute directly to the error in the Z-direction (E_Z). The other identified errors (E_{BX}, E_{BC} and $E_{C\perp X}$) are angular errors that only contribute to Z-direction error when amplified over a lever arm, as was illustrated in Figure 3. These errors are converted to Z-direction errors by multiplying them by the appropriate lever arm distance as follows:

$$\begin{aligned} E_{ZBX} &= \text{Error in the Z-direction caused by } E_{BX} \\ &\approx (E_{BX})(D/2) \\ &\approx (30\ \mu\text{rad})(0.15\ \text{m}) \\ &\approx 4.5\ \mu\text{m} \end{aligned} \tag{3}$$

$$\begin{aligned} E_{ZBC} &= \textit{Error in the Z-direction caused by } E_{BC} \\ &\approx (E_{BC})(D/2) \\ &\approx (15\ \mu\text{rad})(0.15\ \text{m}) \\ &\approx 2.25\ \mu\text{m} \end{aligned} \tag{4}$$

$$\begin{aligned} E_{ZC\perp X} &= \textit{Error in the Z-direction caused by } E_{C\perp X} \\ &\approx (E_{C\perp X})(D/2) \\ &\approx (25\ \mu\text{rad})(0.15\ \text{m}) \\ &\approx 3.75\ \mu\text{m} \end{aligned} \tag{5}$$

The total estimated error in the Z-direction can now be written as:

$$\begin{aligned} E_Z &= \sqrt{E_{ZX}^{\ 2} + E_{ZBX}^{\ 2} + E_{ZC}^{\ 2} + E_{ZBC}^{\ 2} + E_{ZC\perp X}^{\ 2}} \\ &= \sqrt{4\mu m^2 + 4.5\mu m^2 + 2\mu m^2 + 2.25\mu m^2 + 3.75\mu m^2} \\ &= 7.71\ \mu m \end{aligned} \tag{6}$$

In this particular example, the offsets in the Z-direction (Z_1, Z_2 and Z_3) did not factor into the error budget. However, if this was a laser-cutting process that was sensitive to X- and Y-direction errors, those offsets would have mattered.

While the aforementioned method shows ways to estimate the errors that will appear in a particular process, it still represents an *estimated* value of the total error. Extending this concept further, the error values can also be functions or values that change as a function of position. The same approach can be used to get a more accurate representation of the error values over the working volume of the motion system. However, due to the extensive calculations, a purpose-built machine model built using Python™ or Matlab® is typically required. Slocum (1992) presented an effective method using homogeneous transformation matrices (HTM) to model machine errors and perform error budgeting of multi-axis systems. This approach can be used for simple to very complex multi-axis systems.

The example presented in this article is simple in that it only involves two axes and five errors contributing to the system error. Modern machines often use many more axes, and the axis arrangement can get very complex. This approach can be extended to machines with many axes; however, care must be taken in the "accounting" of the errors.

Are There Ways to Minimize Errors?

In this example, the total estimated error in the Z-direction is 7.71 µm. Depending on the desired part tolerances, this error may be too large. In that case, the system machine designer must revisit the design in order to reduce the total error. The error budget allows the designer to experiment with "what if" scenarios to determine where potential improvements can be made. Using different motion axes with smaller errors or aligning the system to tighter tolerances may be necessary. The axis arrangement and machine configuration may need to be revisited in order to determine a better geometric arrangement to accomplish the desired end target error.

If the errors are repeatable, error mapping by measuring errors with an independent measurement device like a laser interferometer may be appropriate. In certain designs, putting a measurement device as close to the functional point as possible may allow for in-situ measurement and correction of errors.

How Can the Estimated Performance Be Verified?

Modern metrology equipment has continued to evolve, and many options exist for machine designers to measure system performance. Still, true functional point measurement can prove very difficult and involved to perform for complex multi-axis systems. For inspection applications, measurement performance is often verified by measuring a known artifact or known good part. For machining or other manufacturing processes, oftentimes parts are machined and then measured using external measurement tools to verify the performance.

However, it is critical to start with approaches and tools that allow the designer to estimate the system-level performance. Capturing more details in the initial error budget estimate increases the machine designer's chances for success at delivering the necessary performance in the final design.

Conclusions

Understanding a motion system's application, configuration and associated component errors allows for a reasonable estimation of system-level errors. Careful component error accounting, coupled with a multi-axis error budget, provides system designers with useful information that can be used to improve system performance. The method shown can be applied to a variety of systems, from simple to complex, as long as proper bookkeeping of the component errors and directions is performed.

References

Slocum, A. (1992). *Precision Machine Design*. Society of Manufacturing Engineers.

About the Author

John Lindell is an Aerotech applications engineering manager with 22 years of precision motion control experience. He is passionate about understanding, developing and deploying high-accuracy motion systems for test and inspection applications. John holds a bachelor's degree in mechanical engineering from Penn State and a master's degree in business administration from the University of Pittsburgh.